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Effects of General Atmospheric Perturbations on Small Debris Particle Orbits

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Atmospheric perturbations due to aerodynamic lift and orthogonal force were included in the Gaussian form of the variational equations. These additional perturbations are relevant to orbital objects with a high ratio of surface area to mass, especially flat particles. Equations for the orbital elements were averaged and expanded in a small eccentricity. The analytical formulation in this paper is applicable to the general case of orbiting aerodynamic bodies, whereas the numerical results emphasize flat particles. The results indicate that although the drag is the dominant force, the nondissipative additional forces cannot be neglected. The additional perturbations mostly contribute periodic terms in addition to secular terms, especially in eccentric orbits and in the case of periodic aerodynamic coefficients.

Nomenclature

A = particle reference area
a = semimajor axis
C_D = drag coefficient
C_L = lift coefficient
C_W = orthogonal force coefficient
E = eccentric anomaly
e = eccentricity
G = angular momentum
H = atmospheric density scale height
i = inclination
M = mean anomaly
m = particle mass
N = normal force
n = mean orbital rate
R = radial force
$\boldsymbol{r}$ = radius vector
S = transverse force
T = tangential force
t = time
u = argument of latitude
V = speed
v = true anomaly
W = orthogonal force
γ = flight path angle
μ = geocentric gravitational constant
ρ = atmospheric density
Ω = longitude of ascending node
ω = argument of perigee

Introduction

THE only atmospheric force considered for most orbital objects is drag. It presents dissipation and is responsible for orbital decay. This paper also considers two other aerodynamic forces: the lift, which is in the orbital plane perpendicular to the velocity; and the orthogonal force. These forces traditionally have been known as "side" forces. However, in this paper we refer to them as "additional" forces.

A typical drag-decayed orbit consists of two phases—circularization, in which the orbit decays to a form of an instantaneously circular orbit, and a spiral orbit directed toward Earth. Numerous publications, mostly from the 1960s, deal with the problem of atmospheric drag. This has been discussed and summarized extensively in the literature.[1–3] The effect of the orbital lift on the perigee motion in the first order of eccentricity is discussed in Ref. 4. However, more comprehensive studies are needed, specifically studies that include all the orbital elements and provide solutions for a higher order of eccentricity. Inclusion of the additional forces is essential when considering bodies with irregular shapes, especially small particles with a high ratio of surface area to mass, because the ratio of lift to drag for a convex body usually increases as the ratio of surface area to mass increases. The minimal ratio corresponds to a sphere, in which case no additional forces are produced. In addition, the acceleration due to the aerodynamic forces is inversely proportional to typical length of the body. The first example that comes to mind is the problem of space debris. The extreme case would be a thin, flat body. The aerodynamics of an idealized disk-shaped body were evaluated[5] and are summarized in Appendix A, which shows that both the drag and the lift have the same order of magnitude; hence, the lift should not be ignored.

In this paper, we consider the contribution of the additional forces on the particle orbit. First, the equations of motion are written in terms of the aerodynamic coefficients. These are expressed in terms of the classical orbital elements in order to gain the greatest physical insight. Then, a discussion of some qualitative properties based on these equations is presented. Next, the averaged orbital elements are evaluated and simplified by assuming a small eccentricity. Finally, the influence of the additional forces is illustrated in numerical examples.

Basic Equations

The perturbations caused by atmospheric forces are naturally expressed in the normal-tangential-orthogonal group of forces (N, T, W). Here, N is directed toward the inside of the orbit; T is along the velocity; and W points to the angular momentum direction. These are related to the atmospheric forces as follows:

$$T = -\frac{1}{2}\frac{A}{m}\rho V^2 C_D \equiv -\text{drag force/mass} \tag{1a}$$

$$N = -\frac{1}{2}\frac{A}{m}\rho V^2 C_L \equiv -\text{lift force/mass} \tag{1b}$$

$$W = -\frac{1}{2}\frac{A}{m}\rho V^2 C_W \equiv -\text{orthogonal force/mass} \tag{1c}$$

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The aerodynamic coefficients C_D, C_L, and C_W depend on the rotational orientation of the body and are formulated in Appendix A. Projecting T, N, and W perturbations into the radial-transverse-orthogonal frame of reference, using the following transformation:

$$\begin{pmatrix} R \\ S \\ W \end{pmatrix} = \begin{pmatrix} \sin\gamma & -\cos\gamma & 0 \\ \cos\gamma & \sin\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} T \\ N \\ W \end{pmatrix} \tag{2}$$

where the flight path angle is a function of the true anomaly

$$\sin\gamma = \frac{e\,\sin v}{\sqrt{1+2e\,\cos v + e^2}}$$
$$\cos\gamma = \frac{1+e\cos v}{\sqrt{1+2e\cos v+e^2}} \tag{3}$$

Substituting R, S, and W into the Gaussian form of the variational equations gives the following equations for the classical orbital elements $(a, e, M_0, i, \omega, \Omega)$:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -B(v)\frac{f_1(v)}{n\sqrt{1-e^2}}C_D$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = -B(v)\frac{\sqrt{1-e^2}}{naf_1(v)}\left[2C_D\,(e+\cos v) - C_L\frac{(1-e^2)\sin v}{1+e\cos v}\right]$$

$$\frac{\mathrm{d}M_0}{\mathrm{d}t} = -B(v)\frac{1-e^2}{naef_1(v)(1+e\cos v)}$$
$$\times\left[2C_D\,(1+e^2+e\cos v)\sin v + C_L\,(1-e^2)\cos v\right]$$

$$\frac{\mathrm{d}i}{\mathrm{d}t} = -B(v)\frac{\sqrt{1-e^2}\cos u}{na(1+e\cos v)}C_W$$

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = -B(v)\,\frac{\sqrt{1-e^2}}{nae}\left[C_D\,\frac{2\sin v}{f_1(v)} + C_L\,\frac{f_2(v)}{(1+e\cos v)f_1(v)}\right.$$
$$\left. + C_W\,\frac{e\sin u\cot i}{1+e\cos v}\right]$$

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = -B(v)\,\frac{\sqrt{1-e^2}\sin u}{na\,(1+e\,\cos v)\sin i}C_W \tag{4}$$

The functions f_1, f_2, and B are defined as

$$f_1 \overset{\text{def}}{=} (1+2e\cos v+e^2)^{\frac{1}{2}}$$
$$f_2 \overset{\text{def}}{=} \cos v + 2e + e^2\cos v \tag{5a}$$

and

$$B \overset{\text{def}}{=} \frac{A}{m}\rho V^2 \tag{5b}$$

where

$$V = \frac{naf_1(v)}{\sqrt{1-e^2}} \tag{5c}$$

Note that Eq. (4) is different from the equations for constant (N, T, W), because the aerodynamic forces depend on the orbital elements and the anomalies. The aerodynamic coefficients are constants or functions of the anomalies.

A comparison between Eq. (4) and the classical literature indicates six new terms in our general case. All orbital elements, except the semimajor axis, are perturbed by the additional aerodynamic forces. The lift and the orthogonal force are perpendicular to the velocity and so they do not affect the energy; hence, the semimajor axis is perturbed by the drag alone. The additional forces mainly perturb the orbital orientation.

Intermediate Qualitative Conclusions

Some conclusions concerning the nature of the new terms can be drawn from Eq. (4). Table 1 presents the contribution of an arbitrary normal force versus the aerodynamic lift. The main difference is reflected in the dependence on the true anomaly. Only the functions of the true anomaly are presented.

Table 1 Contribution of the lift to the variations

Diff. eq.	In terms of N	In terms of C_L
$\frac{de}{dt}$	$\frac{\sin v}{f_1(v)\,(1+e\cos v)}(-N)$	$\frac{f_1(v)\sin v}{1+e\cos v}C_L$ $= (1+O(e^2))\sin v C_L$
$\frac{d\omega}{dt}$	$-\frac{f_2(v)}{f_1(v)(1+e\cos v)}(-N)$	$-\frac{f_1(v)f_2(v)}{1+e\cos v}C_L$ $= (1+O(e^2))f_2(v)\,C_L$

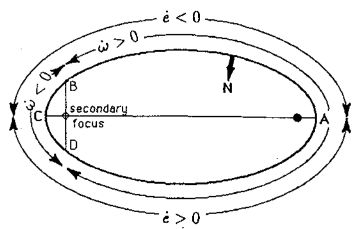

Fig. 1 Contribution of the normal force along an elliptic orbit.

The following discussion is related to Fig. 1, which shows the arcs of equal sign in each contribution related to lift. The general contribution of a normal force to the eccentricity is as follows.[6] A positive value decreases the eccentricity during the first half-revolution and increases it during the second. In addition, the change is greater when the satellite is near the apogee than when it is near the perigee. Only the first statement is valid in the case of aerodynamic lift. The physical reason is that the lift dependence on the velocity increases the contribution near the perigee. The general contribution of the normal force to the line of apsides is a forward rotation when the satellite is in region DAB, and backward rotation in BCD. In the case of lift, the contribution at points B, D is also zero, because in both cases $\cos v + 2e + e^2\cos v = 0$ at those points. In addition, the effectiveness of the force is shifted toward the perigee.

Next, we consider some characteristics of the perturbed orbit. Any elliptic orbit under the influence of the atmospheric drag is passing through the circularization and spiral decay phases. The eccentricity decreases during the first phase and becomes momentarily zero at the beginning of the second phase. We may gain some insight into the influence of the additional forces near the singularity by inspecting and manipulating Eq. (4).

We start by investigating the influence of the lift on the behavior of the eccentricity during the circularization. The orthogonal force does not play any role in this process. An interesting phenomenon is the "cusp" in $e(t)$ as $e \to 0$. It can be detected by the second derivative

$$\frac{\mathrm{d}^2e}{\mathrm{d}t^2} = \frac{\partial\dot e}{\partial a}\dot a + \frac{\partial\dot e}{\partial e}\dot e + \frac{\partial\dot e}{\partial v}\dot v \tag{6}$$

It is easy to see that the partials behave as follows:

$$\left(\frac{\partial\dot e}{\partial a}, \frac{\partial\dot e}{\partial e}, \frac{\partial\dot e}{\partial v}, \dot a, \dot e\right) = O(1) \quad \text{as } e \to 0 \tag{7a}$$

but

$$\dot v = O(1/e) \quad \text{as } e \to 0 \tag{7b}$$

therefore

$$\frac{\mathrm{d}^2e}{\mathrm{d}t^2} = O\left(\frac{1}{e}\right) \quad \text{as } e \to 0 \tag{8}$$

Hence, the lift does not eliminate the "cusp" in $e(t)$. The physical explanation is that the variations in the energy and the angular momentum are caused by the transverse and the tangential perturbations, respectively. The singularity presents a circular orbit in which the lift does not contribute to the above integrals of motion. Because the eccentricity reflects these integrals, we expect that the "cusp" behavior will not be affected by the lift.

Another interesting variable is the argument of latitude. A straightforward substitution shows that

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \frac{\mathrm{d}\omega}{\mathrm{d}t} + \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\sqrt{\mu p}}{r^2} + W\,\frac{\sqrt{1-e^2}\,\sin u}{na\,(1+e\cos v)}\cot i \qquad (9)$$

That is, the argument of latitude is nonsingular as $e \to 0$ and perturbed by the orthogonal force alone.

The last issue is the influence of the additional forces on the perigee and apogee distances. In the current case, the value of the eccentricity is arbitrary. It is convenient to proceed with the eccentric anomaly E as the independent variable.[7] The final form of the equations of motion for the perigee and apogee distances is

$$\frac{\mathrm{d}r_P}{\mathrm{d}E} = -\frac{A}{m}\rho a^2\left[(1-e)(1-\cos E)\sqrt{\frac{1+e\cos E}{1-e\cos E}}C_D + \frac{1}{2}\sin E\sqrt{(1-e^2)(1-e^2\cos^2 E)}C_L\right]$$

$$\frac{\mathrm{d}r_A}{\mathrm{d}E} = -\frac{A}{m}\rho a^2\left[(1+e)(1+\cos E)\sqrt{\frac{1+e\cos E}{1-e\cos E}}C_D - \frac{1}{2}\sin E\sqrt{(1-e^2)(1-e^2\cos^2 E)}C_L\right] \qquad (10)$$

The additional force here is the lift. Its contribution is periodic for a constant C_L and secular for some cases of periodic C_L. The size and magnitude of the secular terms depend on the period and phase of the coefficient. An ideal case would be a lift coefficient that is proportional to $\sin E$. This gives rise to the fastest changes in the perigee and apogee distances. The more realistic case would be a coefficient that is a function of the time or the true anomaly. In the first case, the coefficient can be expressed as a function of E by substituting the Kepler's equation. In the second case, it may be expanded in a small eccentricity to form a series in eccentric anomaly.

Equations for the Average

The influence of the general atmospheric perturbations in the long term is well approximated by the averaged orbital elements. The crux of averaging is to transform the system into an autonomous one. The averaging operator on the variable χ is

$$\frac{\mathrm{d}\bar{\chi}}{\mathrm{d}t} = \frac{1}{2\pi}\int_0^{2\pi}\frac{\mathrm{d}\chi}{\mathrm{d}t}\,\mathrm{d}M = \frac{n}{2\pi}\int_0^{2\pi}\frac{\mathrm{d}\chi}{\mathrm{d}E}\,\mathrm{d}E \qquad (11)$$

Thus, the independent variable on the righthand side of the equations is eliminated and the equations become autonomous. This form also reflects the changes in the orbital elements over one revolution. Changing the independent variable to E, assumming constant coefficients and averaging, leads to

$$\frac{\mathrm{d}\bar{a}}{\mathrm{d}t} = -\frac{1}{2\pi}\frac{A}{m}na^2\,C_D\int_0^{2\pi}\rho\sqrt{\frac{(1+e\cos E)^3}{1-e\cos E}}\,\mathrm{d}E$$

$$\frac{\mathrm{d}\bar{e}}{\mathrm{d}t} = -\frac{1}{2\pi}\frac{A}{m}na(1-e^2)\left[C_D\int_0^{2\pi}\rho\cos E\sqrt{\frac{1+e\cos E}{1-e\cos E}}\,\mathrm{d}E - \frac{C_L}{2\sqrt{1-e^2}}\int_0^{2\pi}\rho\sin E(1-e\cos E)\,\mathrm{d}E\right]$$

$$\frac{\mathrm{d}\bar{i}}{\mathrm{d}t} = -\frac{1}{4\pi}\frac{A}{m}na\,C_W\left[\frac{\cos\omega}{\sqrt{1-e^2}}\int_0^{2\pi}\rho(1+e\cos E) \times(\cos E - e)\,\mathrm{d}E - \sin\omega\int_0^{2\pi}\rho(1+e\cos E)\sin E\,\mathrm{d}E\right]$$

$$\frac{\mathrm{d}\bar{M}_0}{\mathrm{d}t} = \frac{1}{2\pi}\frac{A}{m}\frac{na}{e}\left[C_D\int_0^{2\pi}\rho(1-e^3\cos E)\sin E \times\sqrt{1-e^2\cos^2 E}\,\mathrm{d}E + C_L\sqrt{1-e^2}\int_0^{2\pi}\rho(\cos E - e) \times\sqrt{1-e^2\cos^2 E}\,\mathrm{d}E\right]$$

$$\frac{\mathrm{d}\bar{\omega}}{\mathrm{d}t} = -\frac{1}{2\pi}\frac{A}{m}na\frac{\sqrt{1-e^2}}{e}\Bigg[C_D\int_0^{2\pi}\rho\sin E\sqrt{\frac{1+e\cos E}{1-e\cos E}}\,\mathrm{d}E + \frac{C_L}{2(1-e^2)^{3/2}}\int_0^{2\pi}\rho\Big((1+e^2)(\cos E - e) + 2e(1-e\cos E)\Big)\sqrt{1-e^2\cos^2 E}\,\mathrm{d}E$$
$$- \frac{e\cos i}{2(1-e^2)}C_W\left(\sin\omega\int_0^{2\pi}\rho(\cos E - e)(1+e\cos E)\,\mathrm{d}E + \sqrt{1-e^2}\cos\omega\int_0^{2\pi}\rho\sin E(1+e\cos E)\,\mathrm{d}E\right)\Bigg]$$

$$\frac{\mathrm{d}\bar{\Omega}}{\mathrm{d}t} = -\frac{1}{4\pi}\frac{A}{m}\frac{na}{\sin i}C_W\left[\frac{\sin\omega}{\sqrt{1-e^2}}\int_0^{2\pi}\rho(1+e\cos E) \times(\cos E - e)\,\mathrm{d}E + \cos\omega\int_0^{2\pi}\rho(1+e\cos E)\sin E\,\mathrm{d}E\right] \qquad (12)$$

In the case of variable coefficients, the final discussion of the previous section can be applied here, also.

Although these equations may be numerically solved, an assumption of small eccentricity gives the advantage of approximated simpler equations and much more insight. Modeling the atmospheric density as

$$\rho = \rho_0\exp[-\nu(1-e\cos E)] \qquad (13)$$

where ρ_0 is the density at the perigee and $\nu = \frac{ae}{H}$. Substituting ρ and expanding the integrands as power series in the eccentricity, Eq. (12) yields

$$\frac{\mathrm{d}\bar{a}}{\mathrm{d}t} = -\frac{A}{m}\rho_0 e^{-\nu}na^2C_D\left[I_0 + 2eI_1 + \frac{3}{4}e^2(I_0 - I_2) + \frac{1}{4}e^3(3I_1 + I_3) + \frac{7}{64}e^4(3I_0 + 4I_2 + I_4) + \cdots\right]$$

$$\frac{\mathrm{d}\bar{e}}{\mathrm{d}t} = -\frac{A}{m}\rho_0 e^{-\nu}naC_D\left[I_1 + \frac{1}{2}e(I_0 + I_2) - \frac{1}{8}e^2(5I_1 - I_3) - \frac{1}{16}e^3(5I_0 + 4I_2 - I_4) - \frac{3}{512}e^4(46I_1 + 7I_3 - 5I_5) + \cdots\right]$$

$$\frac{\mathrm{d}\bar{i}}{\mathrm{d}t} = -\frac{1}{2}\frac{A}{m}\rho_0 e^{-\nu}na\cos\omega\,C_W\left[I_1 - \frac{1}{2}e(I_0 - I_2) - \frac{1}{2}e^2 I_1 - \frac{1}{4}e^3(I_0 - I_2) - \frac{1}{8}e^4 I_1 + \cdots\right]$$

$$\frac{\mathrm{d}\bar{M}_0}{\mathrm{d}t} = \frac{A}{m}\rho_0 e^{-\nu}\frac{na}{e}C_L\left[I_1 - eI_0 - \frac{1}{8}e^2(7I_1 + I_3) + \frac{1}{4}e^3 \times(3I_0 + I_2) + \frac{1}{128}(16I_0 - 26I_1 + 16I_2 - 5I_3 - I_5) + \cdots\right]$$

$$
\begin{aligned}
\frac{d\bar{\omega}}{dt} = &-\frac{1}{2}\frac{A}{m}\rho_0 e^{-\nu}\frac{na}{e}\Bigg(C_L\left[I_1 + eI_0 - \frac{1}{8}e^2(3I_1 + I_3)\right.\\
&\left.-\frac{1}{4}e^3(I_0 + I_2) + \frac{1}{128}e^4(10I_1 + 5I_3 + I_5) + \cdots\right]\\
&- e\cos i\sin\omega\, C_W\left[I_1 - \frac{1}{2}e(I_0 - I_2) - \frac{1}{2}e^2 I_1\right.\\
&\left.-\frac{1}{4}e^3(I_0 - I_2) + \cdots\right]\Bigg)\\
\frac{d\bar{\Omega}}{dt} = &-\frac{1}{2}\frac{A}{m}\rho_0 e^{-\nu} na\frac{\sin\omega}{\sin i}C_W\left[I_1 - \frac{1}{2}e(I_0 - I_2) - \frac{1}{2}e^2 I_1\right.\\
&\left.-\frac{1}{4}e^3(I_0 - I_2) - \frac{1}{8}e^4 I_1 + \cdots\right]
\end{aligned}
\tag{14}
$$

Here $I_n = I_n(\nu)$ represents the Bessel function of the first kind and of the order n. Note that the lift was eliminated from the equation for the eccentricity and the drag was eliminated from the equation for the argument of perigee and the mean anomaly at apoch. All the averaged orbital elements are regular (nonsingular) in eccentricity because $I_0(\nu) = O(1)$ and $I_n(\nu) = O(e), n > 0$, as $e \to 0$.

As a qualitative exercise we examine the case of a constant density, where we obtain a great deal of simplification due to the orthogonalization. The straightforward resulting approximations, to order four in eccentricity, are the following:

$$
\begin{aligned}
\frac{d\bar{a}}{dt} &= -\frac{A}{m}\rho na^2 C_D\left[1 + \frac{3}{4}e^2 + \frac{21}{64}e^4 + O(e^6)\right]\\
\frac{d\bar{e}}{dt} &= -\frac{1}{2}\frac{A}{m}\rho nae\, C_D\left[1 - \frac{5}{8}e^2 - \frac{9}{64}e^4 + O(e^6)\right]\\
\frac{d\bar{i}}{dt} &= \frac{1}{4}\frac{A}{m}\rho nae\cos\omega\, C_W\left[1 + \frac{1}{2}e^2 + \frac{3}{8}e^4 + O(e^6)\right]\\
\frac{d\bar{M}_0}{dt} &= -\frac{A}{m}\rho na\, C_L\left[1 - \frac{3}{4}e^2 - \frac{3}{64}e^4 + O(e^6)\right]\\
\frac{d\bar{\omega}}{dt} &= -\frac{1}{2}\frac{A}{m}\rho na\left\{C_L\left[1 - \frac{1}{4}e^2 - \frac{3}{64}e^4 + O(e^6)\right]\right.\\
&\quad\left.+\frac{1}{2}C_W\left[e + \frac{1}{2}e^3 + O(e^5)\right]\cos i\sin\omega\right\}\\
\frac{d\bar{\Omega}}{dt} &= \frac{1}{4}\frac{A}{m}\rho nae\frac{\sin\omega}{\sin i}C_W\left[1 + \frac{1}{2}e^2 + \frac{3}{8}e^4 + O(e^6)\right]
\end{aligned}
\tag{15}
$$

The main advantage of Eq. (14) or Eq. (15) form is the structure of a simple initial value O.D.E., while the solution of Eq. (12) requires the Gaussian quadrature as the numerical integration technique. In addition, the equations for a and e are uncoupled. A mutual division leads to a quadrature. The other equations may be solved by successive approximations or by a straightforward numerical integration. The disadvantage is the restriction to small eccentricity.

The influence of the lift on the eccentricity is averaged out. The geometric reason is the mirror image with respect to the major axis of the ellipse. The physical reason is that the lift does not contribute to the energy, and only its periodic transverse component contributes to the angular momentum. The lift is the dominant contributor to the line of apsides motion because the drag is averaged out. The contribution of the orthogonal force to i, ω, and Ω is in the order of e. This is because it perturbs only the angular momentum in a periodic form with a zero average. Note that the aerodynamic coefficients were taken as constants. The equations for the average are different in the case of time-variant coefficients. For example, in the case of periodic coefficients, we will not obtain the cancellations due to the orthogonalizations, that lead to Eq. (14). This causes some additional secular terms in the orbital elements.

Examples

The following examples demonstrate the effects of the additional forces and give an idea about the order of magnitude of the outcome. Figure 2 shows the normalized Euclidean norm of the deviation from an unperturbed orbit for the various drag, lift, and orthogonal force coefficients. A flat disk was chosen because it represents the extreme ratio of additional force to drag (See Appendix A). The maximum deviation due to the drag + additional forces is found to be 25% greater than the drag-alone deviation.

The perturbed eccentricity vector reflects the influence of the drag and the additional force on the eccentricity and the line of periapsis. Figure 3 presents a numerical solution to the equations for the average. The initial condition is $e = 0.02$ and $\omega = 90$ deg. The drag is responsible for the circularization, whereas the additional forces rotate the line of apsides backward. The influence of the orthogonal force is in order of eccentricity relative to the influence of the lift.

The discussion in this research was based on the assumption that the aerodynamic coefficients are constant. As a motivation for a future research, we demonstrate a case study of a rotating force. Figure 4 illustrates the effect of the frequency of the orthogonal force on the deviation of the inclination in a circular orbit. The phenomenon of resonance at the orbital frequency is shown clearly. Note that a constant force has no effect, and a very high frequency Force is averaged out. The physical reason for the resonance is that a rotation of the orthogonal force in the orbital frequency produces

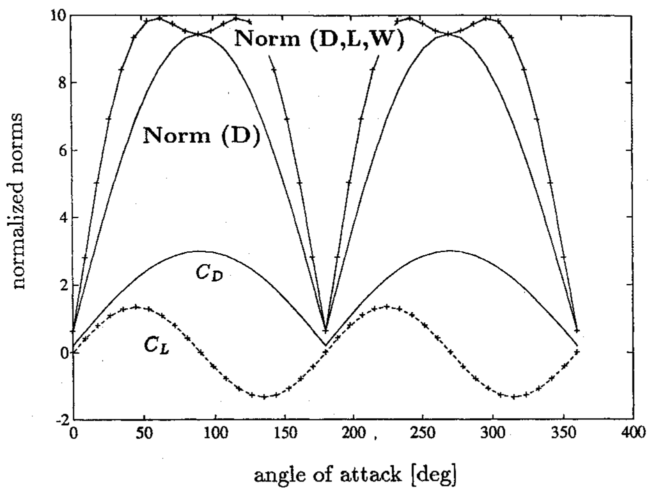

Fig. 2 Deviation norms of the perturbed orbits.

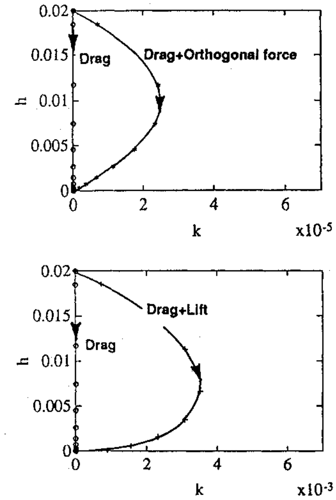

Fig. 3 Influence of the aerodynamic forces on the eccentricity vector.

Table 2 Order of influence of the aerodynamic coefficients

Elements	General	Small eccentricity averaging
a	C_D	$O(1) \cdot C_D$
e	C_D, C_L	$O(e) \cdot C_D$
i	C_W	$O(e) \cdot C_W$
M_0	C_D, C_L	$O(1) \cdot C_L$
ω	C_D, C_L, C_W	$O(1) \cdot C_L + O(e) \cdot C_W$
Ω	C_W	$O(e) \cdot C_W$

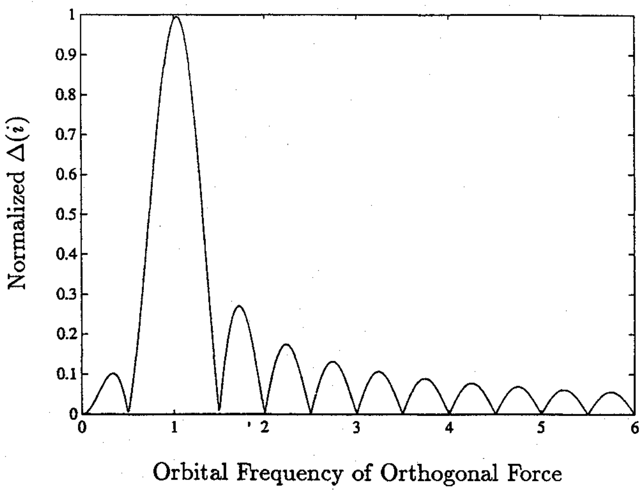

Fig. 4 Effect of the rotating side forces on the inclination.

a torque that is antisymmetrical relative to the node vector. The other extreme case, a constant force, produces an antisymmetrical variation that is averaged out.

Conclusions

This paper considers the aerodynamic additional forces, the lift, and the orthogonal forces on particles orbits. It has been shown that these forces may have a significant influence in some cases. The averaged influence of each aerodynamic coefficient is summarized in Table 2, which presents the coefficients that play a role in the variation of each element, and their relative importance.

It seems that the atmospheric drag is responsible for variations in the shape of the orbit. The lift governs the motion of the perigee, and the orthogonal forces change the orbital orientation. The contribution of the additional forces to the orbit deviation reaches about 25% of the drag-alone contribution. The results examined in this paper point to the need, for further research on the influence of rotating atmospheric perturbations. Although the results can be generalized by accounting for the real atmospheric density, the present model provides an excellent insight into the orbital dynamics.

Appendix A: Flat Disk in a Free Molecular Flow

The free molecular flow is characterized by a large ratio of molecular mean freepath to characteristic body dimension. Because the molecular mean freepath at 200–500 km is on the order of 100 m, the low-orbit particles are certainly in this flow regime. The practical model for the molecule speed is the Maxwell distribution.[8] The molecule speed ratio, which is the ratio between the particle speed to the molecular mean random speed, is also high (>5) at that altitude. The aerodynamic shape that will produce the maximal ratio of additional force to drag is a flat disk. We adopt this shape for methodical reasons to demonstrate the extreme case of the additional forces. The evaluation of the drag and lift was performed[5] and approximated for a high molecular speed ratio. The final approximation is

$$\begin{aligned} C_D &\approx 0.2 + 2.8|\sin\alpha| \\ C_L &\approx 1.1\sin 2\alpha \\ C_W &\approx 1.1\sin 2\beta \end{aligned} \tag{A1}$$

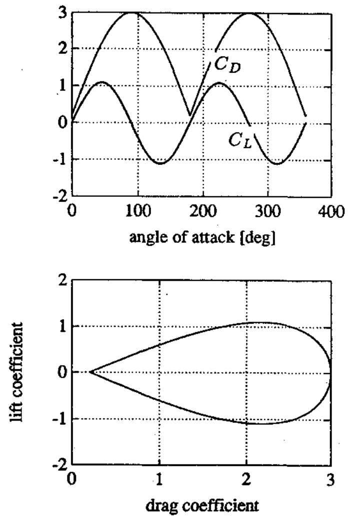

Fig. A1 Drag and lift of a flat disk in a free molecular flow.

where α is the angle of attack: the angle between the surface to the incoming velocity, measured in the orbital plane; and β is the angle of sideslip, measured normal to the orbital plane. These coefficients are shown in Fig A1. The lift reaches almost half the value of the drag in some rotational attitudes of the disk. Note that the lift and the orthogonal force have an identical formulation.

Appendix B: Equations for the Numerical Integration

For the purpose of eliminating the singularity in the eccentricity we need orbital elements that do not depend on the apsidal line. This singularity is eliminated by the substitution $h = e\sin\omega$ and $k = e\cos\omega$, and the new set of orbital elements is (h, k, G, u, i, Ω). The final form of the equations that include the additional aerodynamic perturbations is

$$\begin{aligned} \frac{dh}{dt} &= -2\frac{D}{V}(h + \sin u) \\ &\quad - \frac{L}{V}\frac{(1 - h^2 + k^2)\cos u + 2kh\sin u + 2k}{g_2(u)} \\ &\quad - \frac{W}{V}\frac{kg_1(u)\sin u\cot i}{g_2(u)} \\ \frac{dk}{dt} &= -2\frac{D}{V}(k + \cos u) \\ &\quad + \frac{L}{V}\frac{(1 + h^2 - k^2)\sin u + 2kh\cos u + 2h}{g_2(u)} \\ &\quad - \frac{W}{V}\frac{hg_1(u)\sin u\cot i}{g_2(u)} \\ \frac{dG}{dt} &= -\frac{D}{V}G - \frac{L}{V}\frac{k\sin u - h\cos u}{g_2(u)}G \end{aligned} \tag{B1}$$

$$\begin{aligned} \frac{du}{dt} &= \frac{G}{r^2} + \frac{W}{V}\frac{g_1(u)\sin u\cot i}{g_2(u)} \\ \frac{di}{dt} &= -\frac{W}{V}\frac{g_1(u)\cos u}{g_2(u)} \\ \frac{d\Omega}{dt} &= -\frac{W}{V}\frac{g_1(u)\sin u}{g_2(u)\sin i} \end{aligned}$$

where

$$r = \frac{G^2}{\mu\, g_2(u)} \tag{B2}$$

and

$$V = (\mu/G)g_1(u) \tag{B3}$$

The relations between the aerodynamic forces and the aerodynamic coefficients is

$$\frac{(D, L, W)}{V} = \frac{1}{2}\rho\frac{A}{m}\frac{G}{\mu}\, g_1(u)\,(C_D, C_L, C_W) \tag{B4}$$

were

$$\begin{aligned} g_1 &\stackrel{\text{def}}{=} (1 + 2k\cos u + 2h\sin u + k^2 + h^2)^{\frac{1}{2}} \\ g_2 &\stackrel{\text{def}}{=} 1 + k\cos u + h\sin u \end{aligned} \tag{B5}$$

The essential advantages of these equations are the removal of the singularity due to circular orbits, in such a way that the circularization process can be simulated properly, as well as the pure intergation in time, without solving Kepler's equation. In the case of planar perturbations (drag and lift), the equations may be divided by du/dt and a set of equations in slow variables will be obtained. The disadvantage is the presentation of the singularity due to zero inclination. However, because this paper is concerned only about the atmospheric perturbations, the equatorial orbits are not of special importance for our purposes.

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